

**ABSTRACTS**  
**«Braids and low dimensional topology »**  
**Rennes 31 august – 3 september 2005**

**Mini-courses**

**Alan Reid**

« Arithmetic groups and low dimensional topology »

*These talks will try to give a flavour of why arithmetic hyperbolic orbifolds in low dimensions have been important examples in low dimensional topology. The talks will give definitions, discuss examples as well recent and not so recent work on the geometry and topology of low dimensional arithmetic orbifolds.*

**Andrzej Zuk**

« Random walks on automata groups »

*The class of groups generated by finite automata contains several important examples of infinite groups. In these talks we will present recent developments in this theory with particular emphasis on the study of the random walk operator on these groups. We will explain how results about random walks enabled us to solve some well known problems concerning geometry, analysis and probability on groups. We will concentrate on growth, amenability, spectra of graphs, dynamics of rational maps, Ihara zeta functions and  $L^2$  Betti numbers.*

---

**Talks**

**B. Audoux**

"Catégorification d'un raffinement du polynôme de Jones pour les diagrammes d'entrelacs considérés à isotopie "braid-like" près"

*Une isotopie "braid-like" de diagrammes d'entrelacs est une isotopie n'utilisant que les mouvements de Reidemeister correspondant aux singularités apparaissant lors d'isotopies de diagrammes de tresses. Nous présentons un raffinement du polynôme de Jones, ainsi que sa catégorification, qui ne soit, en général, invariant que par les isotopies "braid-like".*

---

**P. Bellingeri**

"Lower central series for surface braids"

*Let  $G$  be a group and let  $\Gamma_i(G)$  be the  $i$ -th term of the lower central series of  $G$ . Let  $B_n$  be the  $n$ -th Artin braid group. It is easy to prove that  $\Gamma_2(B_n) =$*

$\Gamma_3(B_n)$ . On the other hand it is well known that the pure braid group  $P_n$  is residually nilpotent, i.e.  $\bigcap_{i \geq 1} \Gamma_i(P_n) = 1$ .

Braid groups of surfaces are the fundamental groups of the configuration spaces of distinct points on a surface. They are a common generalization of the Artin braid groups (corresponding to the case where the surface in question is a disc) and of the fundamental groups of surfaces (corresponding to the case when we consider configurations of 1 point). We denote the braid group on  $n$  strands of the surface  $\Sigma$  by  $B_n(\Sigma)$  and we define the pure braid group on  $n$  strands of  $\Sigma$ ,  $P_n(\Sigma)$ , as the kernel of the canonical projection of  $B_n(\Sigma)$  onto the symmetric group  $S_n$ .

Let  $\Sigma$  be a closed oriented surface and let  $n$  be greater than 2. We show that  $P_n(\Sigma)$  is residually nilpotent and that  $\Gamma_3(B_n(\Sigma)) = \Gamma_4(B_n(\Sigma))$ . Moreover, we show that the quotient group  $\Gamma_2(B_n(\Sigma)) / \Gamma_3(B_n(\Sigma))$  is a (non trivial) cyclic group of finite order depending on the number of strands and the genus of  $\Sigma$ . We give some partial results for the case of surface braid groups on 2 strands. In particular we prove that the braid group on 2 strands of the torus is residually nilpotent. The results have been obtained in collaboration with J. Guaschi (Toulouse) and S. Gervais (Nantes).

---

## J. Burillo

"The pure braided Thompson's group"

After the introduction by Brin and Dehornoy of the braided Thompson's group, we will study its subgroup of pure braided tree pairs. We will prove that the subgroup of pure braided tree pairs is finitely presented and show an explicit presentation. We will consider a different presentation of the whole braided Thompson's group different that that of Brin.

---

## R. Fenn

"Using Generalised Quaternions to find Invariants of Virtual Knots"

Solutions of the non-commuting equation  $A'B'AB-BA'B'A=B'AB-A$  where  $X'$  is the inverse of  $X$ , can be used to find invariants of virtual knots and representations of the virtual braid group. All quaternion solutions have been found. Now all  $2 \times 2$  matrix solutions are known. This talk will describe the solutions and their applications.

---

## T. Kuessner

"Generalization of Agol's inequality"

We derive a general lower bound for the normal Gromov norm of a lamination in terms of the topology of the complementary regions. In particular we get that hyperbolic 3-manifolds of volume smaller than  $2.02..$  (e.g. hyperbolic manifolds obtained by Dehn filling the figure eight knot complement) can not carry tight laminations such that the complementary regions of the lamination decompose into  $I$ -bundles and solid tori.

## **J. Mairesse**

### "Randomly Growing Braid on Three Strands and the Manta Ray"

*Consider the braid group on three strands  $B_3 = \langle a, b \mid aba = bab \rangle$ . A random braid is formed by repeatedly choosing, randomly and independently, one of the generators  $\{a, a^{-1}, b, b^{-1}\}$  according to some prescribed probability distribution  $\mu$ . The goal is to compute the asymptotic growth rate of the random braid (the drift of the random walk) as a function of  $\mu$ . We achieve it by explicitly describing the "harmonic measure" (which gives the "direction" of escape to infinity). This measure is obtained by composing a "Markovian multiplicative" measure with a rational transduction. Dans cet expose, j'utilise des resultats qui sont expliques dans un expose propose par F. Matheus.*

---

## **G. Masbaum**

### "Perturbative expansion of mapping class group representations in TQFT"

*We show that the integral  $SO(3)$ -TQFT's studied in previous joint work with Pat Gilmer have a perturbative expansion as the order of the root of unity goes to infinity. We obtain a new 'universal' representation of the Torelli group which gives a TQFT interpretation of Ohtsuki's power series invariant of homology spheres. As a byproduct, we obtain a purely skein-theoretical construction of this invariant.*

---

## **G. Massuyeau**

### "Some finiteness properties for the Reidemeister-Turaev torsion of three-manifolds"

*We prove some finiteness properties for the Reidemeister-Turaev torsion of closed oriented three-manifolds with respect to Torelli surgeries, which are cut-and-paste operations preserving the homology type of the manifolds. In general, those properties require the manifolds to come equipped with an Euler structure and a homological parametrization.*

---

## **F. Mathéus**

### "Growth series for Artin groups of dihedral type"

*We consider the Artin group of dihedral type defined by the presentation  $A_k = \langle a, b \mid aba\dots = bab\dots \rangle$  with  $k$  terms in both terms of the relation. We prove that the spherical growth series and the geodesic growth series of  $A_k$  with respect to the Artin generators  $\{a, b, a^{-1}, b^{-1}\}$  are rational. We provide explicit formulas for the series. This is joint work with Jean Mairesse.*

---

## **T. Morifuji**

"von Neumann rho-invariant, first Morita-Mumford class and Rochlin invariant"

*For a surface bundle over the circle, we describe a relation among the von Neumann rho-invariant, the first Morita-Mumford class and the Rochlin invariant in the framework of bounded cohomology*

---

## **L. Paris**

"La conjecture de Makanin est vraiment fausse pour les mapping class groups."

*Makanin avait conjecturé que si deux tresses avaient un exposant commun de même ordre, alors celles-ci étaient conjuguées. Ce fait a d'ailleurs été démontré par Juan Gonzalez-Meneses en utilisant des arguments géométriques. Dans cet exposé nous présenterons les limites d'une généralisation de ce résultat aux mapping class groups. Pour connaître la réponse il vous faudra suivre l'exposé. Je tiens toutefois à signaler que ceci est un travail en collaboration avec Christian Bonatti*

---

## **M. Wolff**

"Framed holonomic knots"

*A holonomic knot is a knot in 3-space which arises as the 2-jet extension of a smooth function on the circle. A holonomic knot associated to a generic function is naturally framed by the blackboard framing of the knot diagram associated to the 1-jet extension of the function. There are two classical invariants of framed knot diagrams: the Whitney index (rotation number)  $W$  and the self linking number  $S$ . For a framed holonomic knot we show that  $W$  is bounded above by the negative of the braid index of the knot, and that the sum of  $W$  and  $|S|$  is bounded by the negative of the Euler characteristic of any Seifert surface of the knot. The invariant  $S$  restricted to framed holonomic knots with  $W=m$ , is proved to split into  $n$ , where  $n$  is the largest natural number with  $2n < |m|+1$ , integer invariants. Using this, the framed holonomic isotopy classification of framed holonomic knots is shown to be more refined than the regular isotopy classification of their diagrams.*

---