

Recommendations for your notes

Here are some suggestions that can help you to organize your notes when reading the papers for the Belle Ile meeting related to the *Source book in real algebraic geometry* project.

Required notions and themes

We recommend to associate to the paper you are reading some kind of meta-informations : the REQUIRED NOTIONS and the THEMES related to this paper.

NOTIONS REQUIRED :

Def. : List of the notions required to understand the text.

Expl. : These notions help the reader to select the texts he is able to read or to evaluate the notions he has to learn before his reading. Ideally these notions could be explained in a glossary.

Examples of mathematical required notions (this list has to be completed by you) :

Irreducible component of algebraic sets, Hilbert basis theorem, Real closed field, First order formula, Quantifier elimination, Analytical variety, etc.

Examples of historiographical required notions (tbc) : arithmetisation, lecture, modernity, etc.

THEMES :

Def. : List of the themes to which the text is related.

Expl. : People interested in those themes could find interest in the text. The complete list of the texts related to one theme could be easily produced.

Examples of mathematical themes :

Abstract Algebra, Algebraic Geometry, Algebraic Topology, Algorithms, Cauchy Index, Commutative Algebra, Complexity, Connected Components, Critical Points Method, Cylindrical Decomposition Decision Methods, Differential Topology, Elimination, Existential Theory of the Reals, Model Theory, Nash Functions, Positivstellensatz, Quadratic Forms, Quantifier Elimination, Quantitative Aspects, Real Algebra, Real Algebraic Geometry, Real Algebraic Variety, Real Analytic Geometry, Real Root Counting, Resolution of Systems of Equations, Semi-algebraic Geometry, Semi-analytic Geometry, Singularities, Stratifications, Sums of Squares, Topology of Real Algebraic Curves, Topology of Real Algebraic Sets, Triangulations, Undecidability, Valuation, Zorn's Lemma (this list is a subset of a list of key words for real algebraic geometry proposed by Savvas Perikleous and Marie-Françoise Roy for a bibliographical project in real algebraic geometry, these themes look relevant for the papers discussed in Belle Ile, they have to be completed by you)

Examples of historiographical themes :

Abstraction, Application, Axiomatics, Cultural transfer, Effectivity, Emergence of a discipline, Figures and geometrical representations, Institutions, International relations, Justifications, Lecture, Logic and mathematics, Mathematical interdisciplinarity, Modern algebra, Modernity, Set theoretic mathematics, Structures, Teaching, etc.

Note that the process of attaching REQUIRED NOTIONS and THEMES to a whole paper is a first step, it will be also necessary to attach REQUIRED NOTIONS and THEMES to portions of the paper, and also to your own notes. For example, if reading your notes necessitates from the reader some mathematical or historiographical prerequisites.

Notes

A note is typically something written by you about a text, which is in general a precise portion of a paper (or the whole paper).

We propose to assign to a note one or several types, according to its nature.

TYPES

The types we propose are :

explanation
rereading
commentary
summary
question
reference
note

explanation : gives some mathematical explanations or complements using notations, definitions, notions, etc. of the author of the paper. An explanation is a text that could have been written by the author of the paper.

rereading : gives some mathematical explanations or complements using notations, definitions, notions, etc. different from those of the author of the paper. It is a text reflecting how somebody else (for instance, you or a mathematician of our corpus) understands the paper. There could be several rereadings (if possible, indicate who is the author of the rereading).

commentary : the note consists of some remarks on the text which are neither explanation nor rereading.

summary : the note summarizes the text.

question : the note identifies some problems with some assertions in the text.

reference : the note contains details on the reference to which the text refers.

note : type to be used when the others are not correct.

Remarks :

- With these conventions footnotes are eliminated.
- When a text is included in an other the notions required and the themes are inherited and don't need to be repeated.

Examples of Required Notions and THEMES

They are very sketchy and are only meant to illustrate what we have in mind.

Example 1.

For the paper of Tarski

REQUIRED NOTIONS :

- (math.) First order logic, real closed field, quantifier elimination (tbc)
- (hist.) (tbc)

THEMES :

- (math.) Algorithms, Complexity, Model Theory, Quantifier Elimination, Quantitative Aspects, Real Root Counting, Semi-algebraic Geometry (tbc)
- (hist.) Axiomatics, Effectivity, Institutions, International relations, Justifications, Lecture, Logic and mathematics, Mathematical interdisciplinarity, Modern algebra (tbc)

Example 2.

For the paper of Whitney

NOTIONS REQUIRED :

- (math.) Ideal, Hilbert basis theorem, Irreducible components of algebraic varieties, Dimension of irreducible algebraic variety (tbc)
- (hist.) (tbc)

THEMES :

- (math.) Commutative Algebra, Connected Components, Differential Topology, Quantitative Aspects, Real Algebraic Variety, Real Analytic Variety, Semi-algebraic Geometry, Semi-analytic Geometry, Singularities, Stratifications, Topology of Real Algebraic Sets, Triangulations (tbc)
- (hist.) Justifications, Mathematical interdisciplinarity (tbc)

Examples of Notes

Example 3.

TYPES : *commentary*

TEXT : « (from Tarski) The importance of the decision problem for the whole of mathematics (and for various special mathematical theories) was stressed by Hilbert, who considered this as the main task of a new field of mathematical research for which he suggested the term "metamathematics".

NOTE : (Alain Herreman) [to be developed: Overview about Hilbert's program and the relation between logic and mathematics from FREGE to TARSKI, with an accent on the shift introduced by TARSKI regarding the relation between metamathematics and mathematics.]

Example 4.

TYPES : *explanation, commentary*

TEXT : «(from Tarski) Hence I was happy when in the beginning of 1948, the RAND Corporation, Santa Monica, California (...) Other, more theoretical aspects of the problems discussed were treated less thoroughly, and only in notes.

NOTE : (Alain Herreman) [Some explanations about the RAND Corporation and some commentaries about the fact that this article is produced in this context.]

Example 5.

TYPES : *reference*

TEXT : « (from Tarski) Löwenheim (1915) gave a decision method for the class of correct formulas of the lower predicate calculus involving only one variable. »

NOTE : (Alain Herreman) text extracted from Löwenheim (1915) THEOREM 5. The evaluation of a Π or Σ over all relatives is always possible for an expression that contains only singularly relative coefficients (or, in addition, at most the coefficients of $1'$ or $0'$) and finitely many Σ or Π over all subscripts. [This is the theorem of Löwenheim as translated in Heijenoort 1967, p. 245]

Example 6.

TEXT : « By a decision method for a class K of sentence (or other expressions) (...) the elementary theory of rings (Mostowski and Tarski), the elementary theories of groups and lattices (Tarski), and the elementary theory of fields (Mrs. Robinson). »

TYPES : *commentary*

NOTE : (Alain Herreman & Catherine Goldstein) [Commentary on the history here produced by Tarski.]

TYPES : *commentary, explanation*

NOTE : (Alain Herreman) [Explanations about decision methods, examples of applications, etc. Decision methods and completeness.]

TYPES : *commentary*

NOTE : (Alain Herreman) [Commentary on decision methods *vs* algorithms.]

Example 7.

TYPES : *note*

TEXT : « (from Tarski) As is well know, any two polygons of equal area, P and Q , can be decomposed into the same finite number of non-overlapping triangles in such a way that each triangle in P is congruent to the corresponding triangle in Q .»

NOTE : (Alain Herreman) related to [Euclide I-45]

Example 8.

TYPES : *commentary*

TEXT : « (Tarski) elementary

NOTE : (Alain Herreman) [to be developed: About the use of «elementary» by TARSKI]

Example 9.

TYPES : *commentary*

TEXT : (Whitney)« elementary »

NOTE : (Marie-Françoise Roy) [to be developed: About the use of «elementary by WHITNEY]

Example 10.

TYPES : *rereading, summary*

TEXT : (Whitney's paper)

NOTE : (Marie-Françoise Roy) The main result of the paper is that every algebraic variety can be decomposed into a union of smooth connected semi-algebraic sets of various dimensions

Example 11.

TYPES : *question*

TEXT : (second paragraph of Whitney's introduction)

NOTE : (Marie-Françoise Roy) Why not use the terminology from differential geometry: manifold, smooth, was not it existing already when the paper was written ?

Example 12.

TYPES : *rereading*

TEXT : (from Hermite 1852) « it is by means of symmetric functions of the roots of the proposed equations that the functions analogous to those of M. Sturm are immediately expressed and the properties of these functions are deduced from their very law of formation » .

NOTE : (Catherine Goldstein) Hermite attributes to Sylvester this rereading of Sturm's original construction.